A new Stata command for computing and graphing percentile shares

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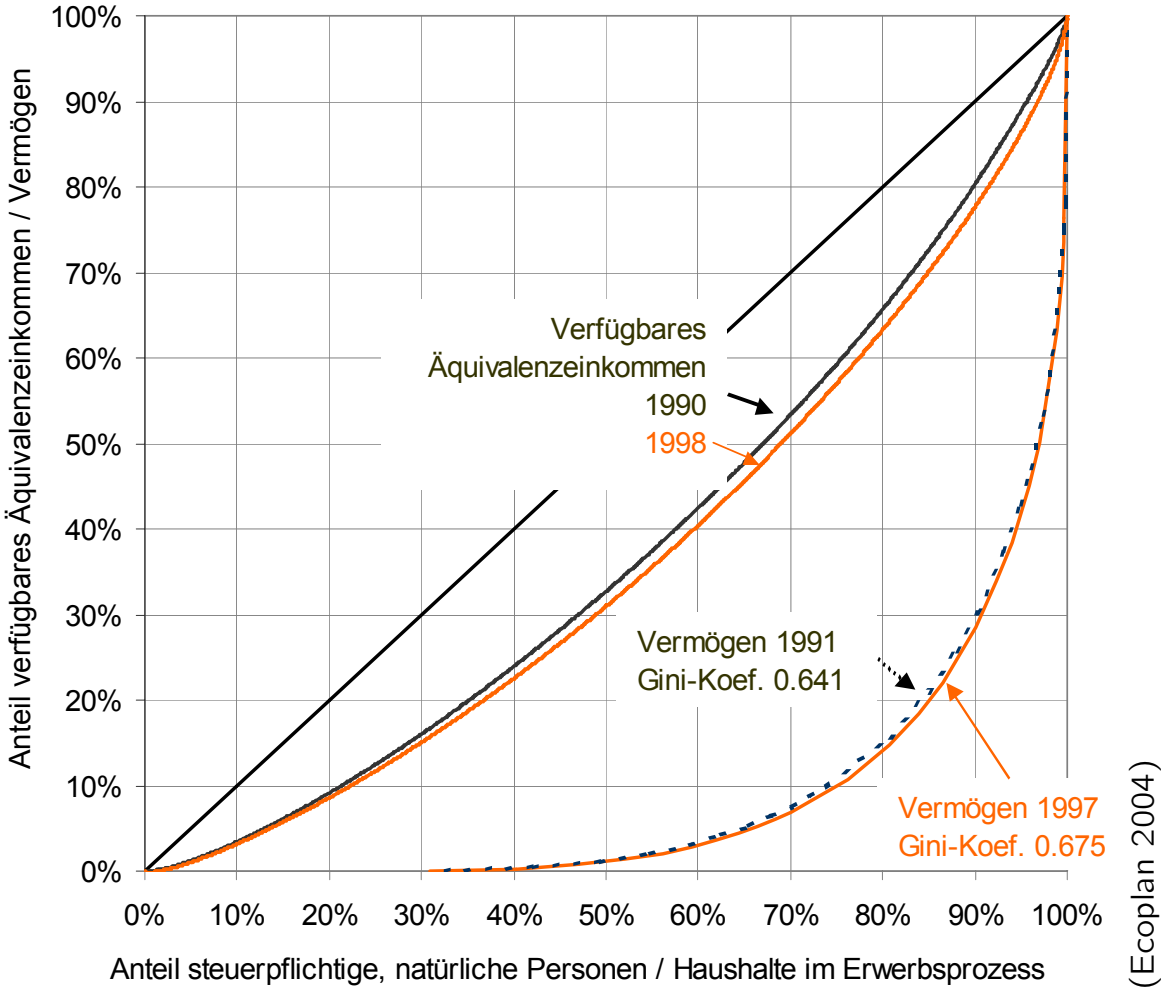


- http://www.youtube.com/watch?v=slTF_XXoKAQ
- https://www.ted.com/talks/dan_ariely_how_equal_do_we_want_the_world_to_be_you_d_be_surprised

Outline

- Motivation
- Estimation of percentile shares
- The `pshare` command
- Examples using Bern tax data

Estimation of percentile shares

- Outcome variable of interest, e.g. income: Y
- Distribution function: $F(y) = \Pr\{Y \leq y\}$
- Quantile function: $Q(p) = F^{-1}(p) = \inf\{y | F(y) \geq p\}$, $p \in [0, 1]$
- Lorenz ordinates:

$$L(p) = \int_{-\infty}^{Q_p} y \, dF(y) \Big/ \int_{-\infty}^{\infty} y \, dF(y)$$

- Finite population form:

$$L(p) = \sum_{i=1}^{N} Y_i \, \mathcal{I}\{Y_i \leq Q_p\} \Big/ \sum_{i=1}^{N} Y_i$$

Estimation of percentile shares

- Percentile share: proportion of total outcome within quantile interval $[Q_{p_1}, Q_{p_2}]$, $p_1 \leq p_2$

$$S(p_1, p_2) = L(p_2) - L(p_1)$$

- Percentile share "density":

$$D(p_1, p_2) = \frac{S(p_1, p_2)}{p_2 - p_1} = \frac{L(p_2) - L(p_1)}{p_2 - p_1}$$

Estimation of percentile shares

- Estimation given sample of size n:

$$\widehat{S}_n(p_1, p_2) = \widehat{L}_n(p_2) - \widehat{L}_n(p_1)$$

$$\widehat{L}_n(p) = (1-\gamma)\widetilde{Y}_j + \gamma\widetilde{Y}_{j+1} \quad \text{where } \widehat{p}_j \leq p < \widehat{p}_{j+1} \text{ with } \widehat{p}_j = \frac{j}{n}$$

$$\widetilde{Y}_j = \sum_{i=1}^{j} Y_{(i)} \Big/ \sum_{i=1}^{n} Y_i \quad \text{where } Y_{(i)} \text{ refers to ordered values}$$

$$\gamma = \frac{p - \widehat{p}_j}{\widehat{p}_{j+1} - \widehat{p}_j} \quad \text{(linear interpolation)}$$

- Standard errors
  - using estimating equations approach as proposed by Binder and Kovacevic (1995)
  - supports complex survey data

The `pshare` command

- `pshare estimate`
  - estimates the percentile shares and their variance matrix
  - arbitrary cutoffs for the percentile groups
  - joint estimation across multiple outcome variables or subpopulations
  - shares as proportions, densities, totals, or averages
  - etc.
- `pshare contrast`
  - computes contrasts between outcome variables or subpopulations
  - differences, ratios, or log ratios
- `pshare stack`
  - displays percentile shares as stacked bar
- `pshare histogram`
  - displays percentile shares as histogram

Examples

```
. use taxdata
(Some tax data)

. describe

Contains data from taxdata.dta
  obs:       119,939                          Some tax data
 vars:             3                          27 Jun 2015 23:49
 size:     1,079,451
-------------------------------------------------------------------------------
              storage   display    value
variable name   type    format     label      variable label
-------------------------------------------------------------------------------
agecat          byte    %9.0g      agecat     Age group
income          float   %9.0g                 Total income
wealth          float   %9.0g                 Net wealth
-------------------------------------------------------------------------------
Sorted by:

. help pshare
```

Quintile shares (the default)

```
. pshare estimate income
Percentile shares (proportion)                    Number of obs     =    119,939
```

income	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
0-20	.029269	.0002685	109.02	0.000	.0287428	.0297952
20-40	.1048592	.0004634	226.30	0.000	.103951	.1057674
40-60	.1645584	.0006001	274.24	0.000	.1633823	.1657345
60-80	.2365146	.0008311	284.59	0.000	.2348856	.2381435
80-100	.4647989	.0018814	247.05	0.000	.4611113	.4684864

Interpretation: The top 20% percent of the population get 46.5% of all income; the bottom 20% only get 2.9% of all income etc.

Decile shares

```
. pshare estimate income, nquantiles(10)
Percentile shares (proportion)                    Number of obs     =    119,939
```

income	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
0-10	.0045258	.0000935	48.42	0.000	.0043426	.004709
10-20	.0247432	.0001932	128.07	0.000	.0243645	.0251219
20-30	.0435825	.0002279	191.23	0.000	.0431359	.0440292
30-40	.0612766	.0002527	242.45	0.000	.0607812	.061772
40-50	.0752199	.0002815	267.19	0.000	.0746681	.0757717
50-60	.0893385	.0003267	273.48	0.000	.0886982	.0899788
60-70	.1065221	.0003835	277.74	0.000	.1057704	.1072739
70-80	.1299924	.0004627	280.95	0.000	.1290855	.1308993
80-90	.1654318	.0005818	284.32	0.000	.1642914	.1665722
90-100	.2993671	.0023598	126.86	0.000	.2947419	.3039922

Bottom 50%, Mid 40%, and Top 10%

```
. pshare estimate income wealth, percentiles(50 90)
Percentile shares (proportion)                    Number of obs     =    119,939
```

	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
income						
0-50	.2093481	.0008937	234.24	0.000	.2075964	.2110998
50-90	.4912848	.0016618	295.64	0.000	.4880278	.4945419
90-100	.2993671	.0023598	126.86	0.000	.2947419	.3039922
wealth						
0-50	-.0237426	.0010954	-21.67	0.000	-.0258896	-.0215956
50-90	.3042619	.0062104	48.99	0.000	.2920896	.3164343
90-100	.7194807	.0057992	124.07	0.000	.7081143	.730847

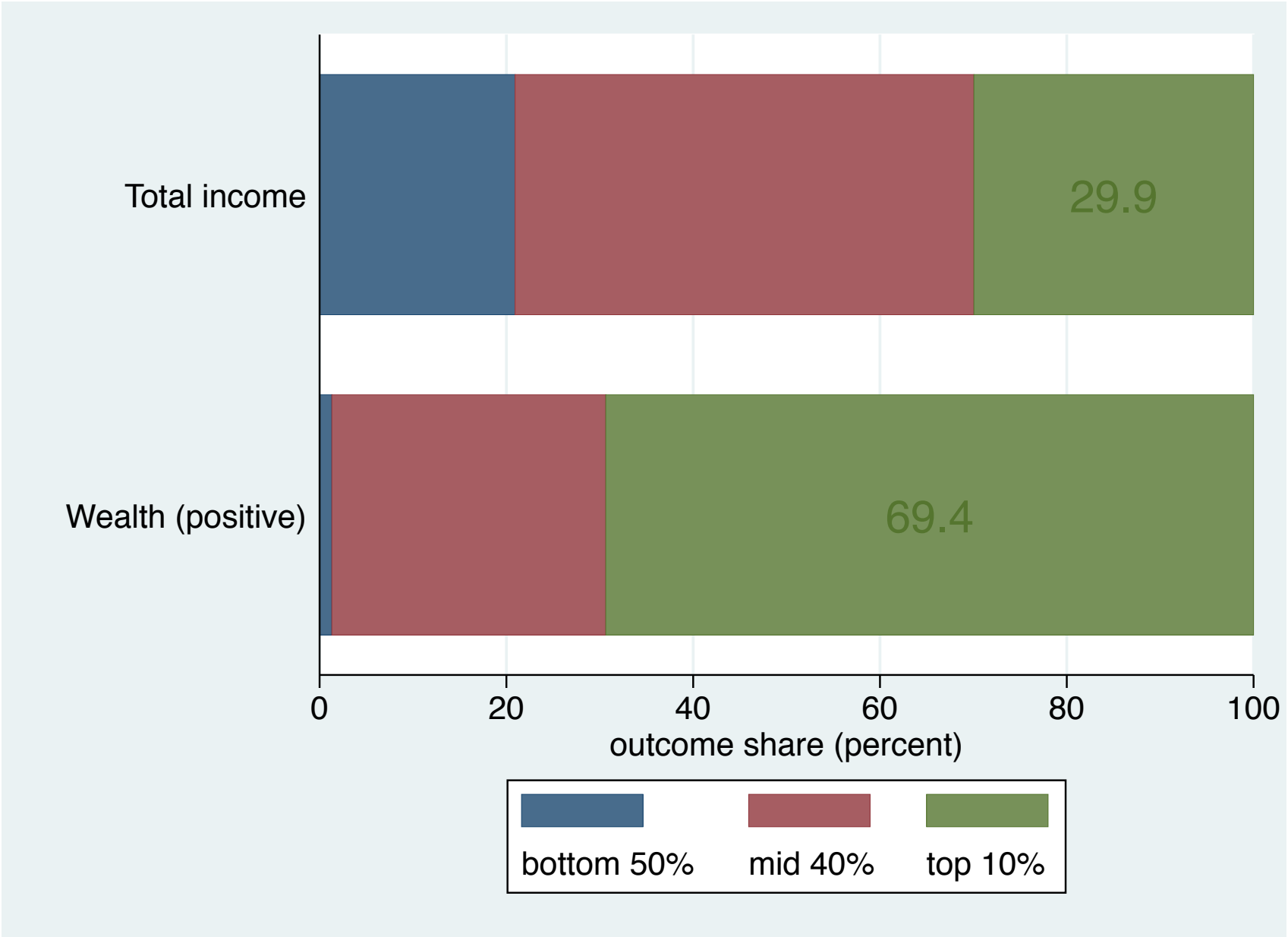
Stacked bars plot

```
. generate wealth0 = cond(wealth<0, 0, wealth)
. label variable wealth0 "Wealth (positive)"
. pshare estimate income wealth0, percentiles(50 90) percent
Percentile shares (percent)                     Number of obs     =    119,939
```

	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
income						
0-50	20.93481	.0893717	234.24	0.000	20.75964	21.10998
50-90	49.12848	.1661772	295.64	0.000	48.80278	49.45419
90-100	29.93671	.2359796	126.86	0.000	29.47419	30.39922
wealth0						
0-50	1.314179	.029754	44.17	0.000	1.255862	1.372496
50-90	29.32997	.5773461	50.80	0.000	28.19838	30.46156
90-100	69.35585	.6038125	114.86	0.000	68.17239	70.53931

```
. pshare stack, plabels("bottom 50%" "mid 40%" "top 10%") ///
>     values mlabsize(zero) p3(mlabsize(large))
```

Stacked bars plot



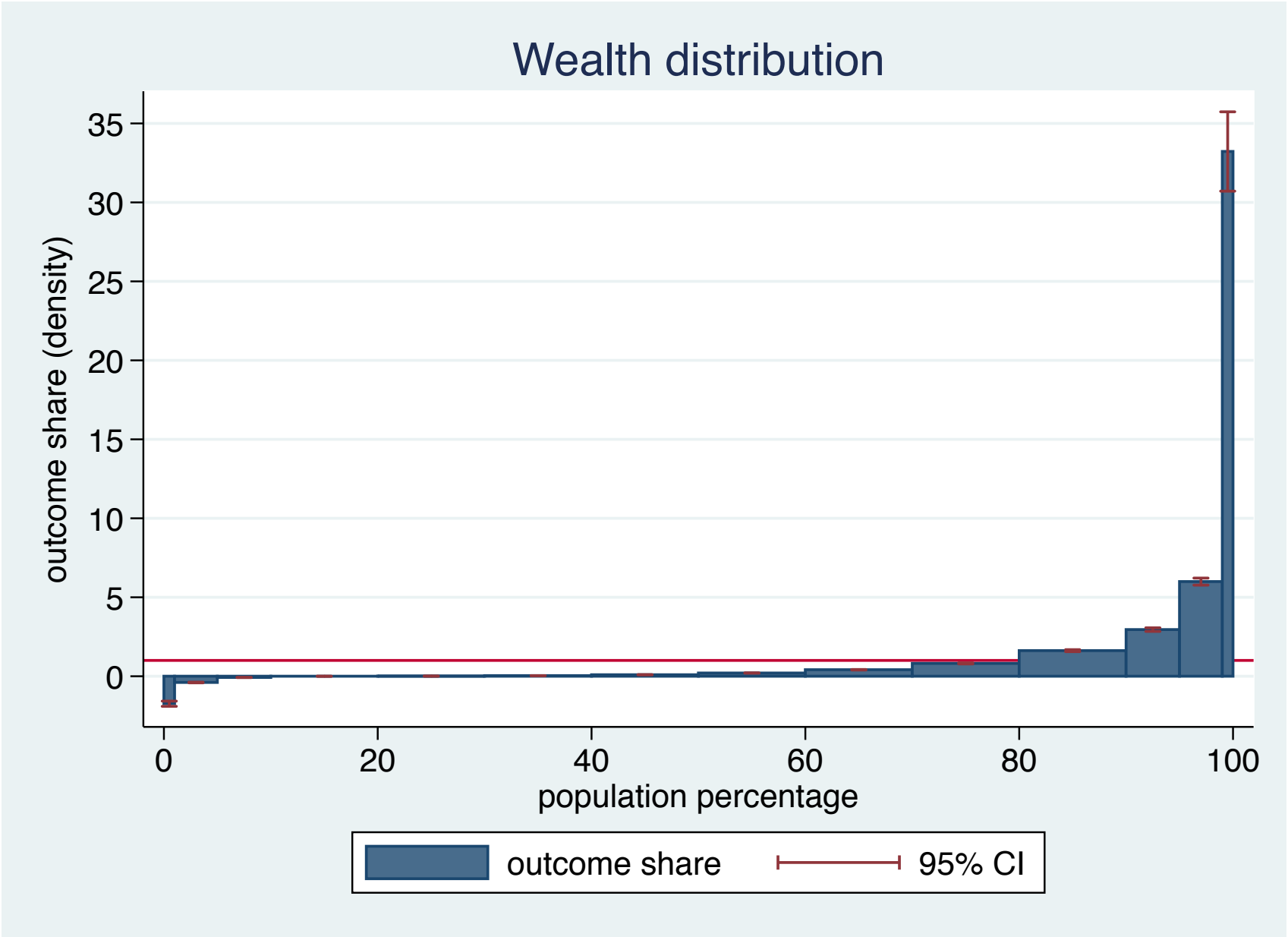
Histogram of densities

```
. pshare estimate wealth, p(1 5 10(10)90 95 99) density
Percentile shares (density)                     Number of obs     =    119,939
```

wealth	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
0-1	-1.740562	.0843581	-20.63	0.000	-1.905903	-1.575222
1-5	-.3963251	.0106253	-37.30	0.000	-.4171505	-.3754998
5-10	-.0789102	.0027628	-28.56	0.000	-.0843253	-.0734951
10-20	-.0017146	.0001861	-9.21	0.000	-.0020794	-.0013497
20-30	.0062028	.0002404	25.80	0.000	.0057316	.0066741
30-40	.0363781	.0008941	40.69	0.000	.0346257	.0381306
40-50	.0937488	.002118	44.26	0.000	.0895975	.0979
50-60	.2023709	.0044304	45.68	0.000	.1936873	.2110544
60-70	.4100251	.0087438	46.89	0.000	.3928874	.4271628
70-80	.811196	.0168879	48.03	0.000	.778096	.8442961
80-90	1.619027	.0328938	49.22	0.000	1.554556	1.683499
90-95	2.951972	.0589668	50.06	0.000	2.836398	3.067546
95-99	5.990715	.114097	52.51	0.000	5.767087	6.214344
99-100	33.22535	1.281602	25.92	0.000	30.71343	35.73727

```
. pshare histogram, yline(1) ylabel(0(5)35, angle(hor)) ti(Wealth distribution)
```

Histogram of densities

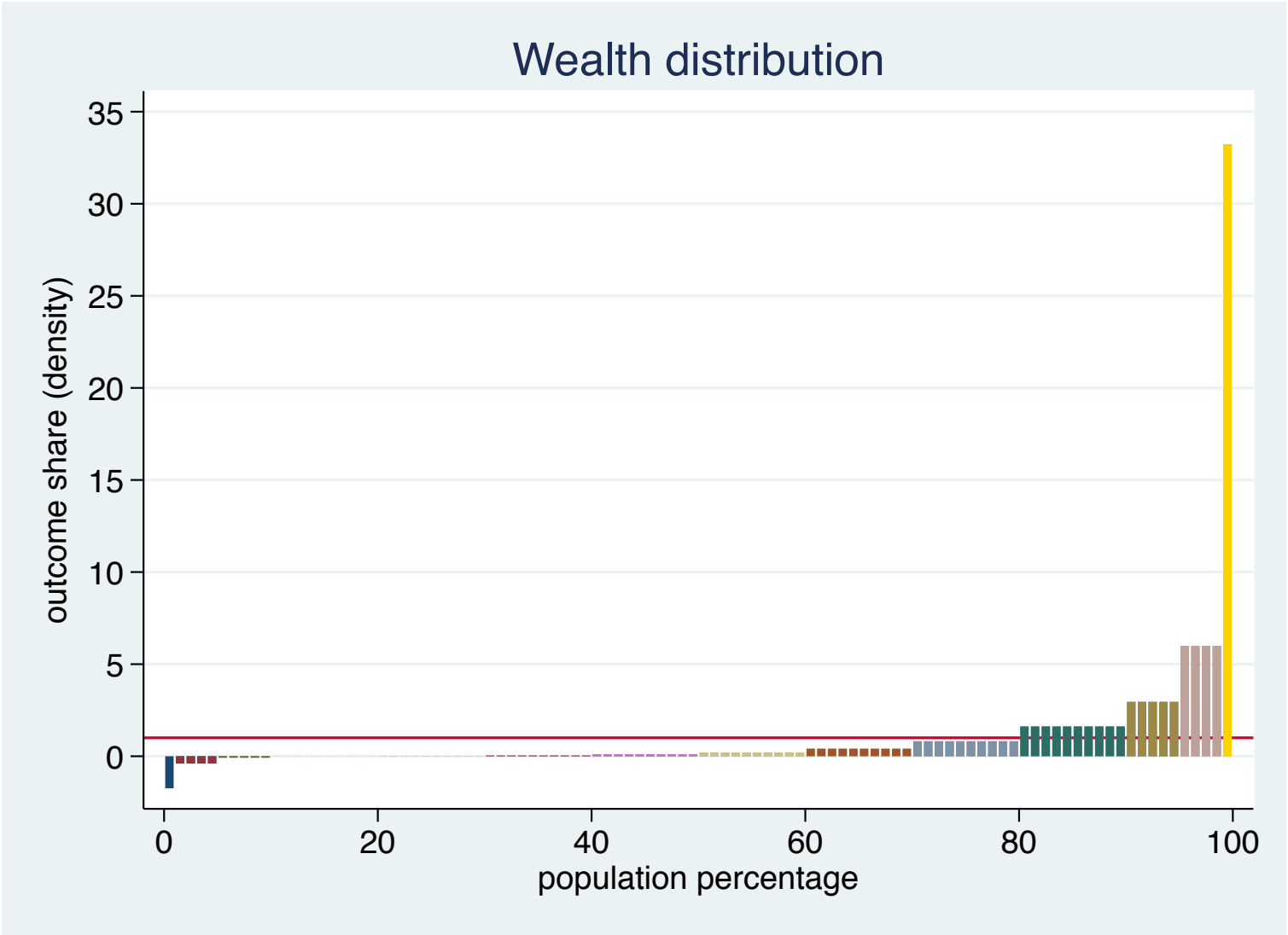


Histogram of densities

- Interpretation
  - Take 100 dollars and divide them among 100 people who line up along the x-axis.
  - The heights of the bars shows you how much each one gets.
  - If all get the same, then everyone would get one dollar (red line).
  - However, according to the observed distribution, the rightmost person (i.e. the richest) would get 33 (!) of the 100 dollars, the next four would get 6 dollar each, and so on.
  - At the bottom, there are also some people you would have to take away some money (e.g., you would have to take away 1.74 dollars from the rightmost person).

Using spikes and group-specific styles

```
. pshare hist, yline(1) ylabel(0(5)35, angle(hor)) ti(Wealth distribution) ///
>     spikes(100) lw(*3) psep legend(off)
```



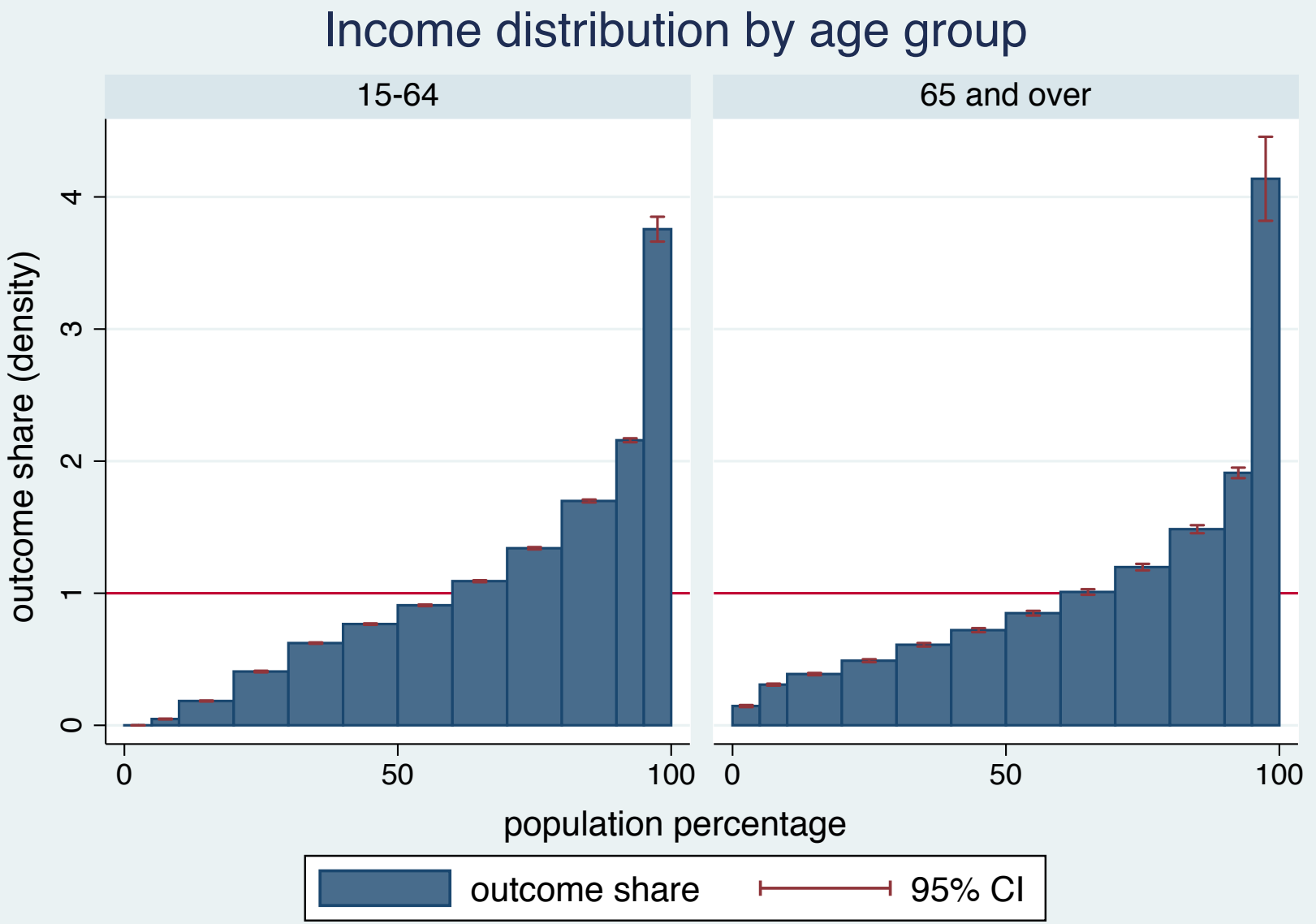
Analysis of subpopulations

```
. pshare estimate income, p(5 10(10)90 95) over(agecat) density
Percentile shares (density)                     Number of obs     =    119,939
          15: agecat = 15-64
          65: agecat = 65 and over
```

income	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
15						
0-5	.0002108	.0000446	4.73	0.000	.0001234	.0002982
5-10	.0477019	.001519	31.40	0.000	.0447247	.050679
10-20	.1839598	.0019449	94.59	0.000	.1801479	.1877717
20-30	.4072382	.0030794	132.25	0.000	.4012026	.4132738
30-40	.62253	.0026311	236.60	0.000	.617373	.627687
40-50	.7665669	.0027358	280.20	0.000	.7612048	.7719291
50-60	.9087896	.0031534	288.20	0.000	.9026091	.9149701
60-70	1.091312	.0037406	291.75	0.000	1.08398	1.098643
70-80	1.340543	.0045022	297.75	0.000	1.331719	1.349367
80-90	1.698174	.0056207	302.13	0.000	1.687158	1.70919
90-95	2.157966	.0076957	280.41	0.000	2.142882	2.173049
95-100	3.755896	.0478808	78.44	0.000	3.66205	3.849741
65						
0-5	.1460041	.0039658	36.82	0.000	.1382311	.153777
5-10	.3081252	.0037889	81.32	0.000	.300699	.3155513
10-20	.3879465	.0043662	88.85	0.000	.3793888	.3965043
20-30	.4893443	.0056362	86.82	0.000	.4782975	.5003911
30-40	.6099115	.006742	90.46	0.000	.5966972	.6231257
40-50	.7204667	.0078164	92.17	0.000	.7051468	.7357867
50-60	.8488989	.0091605	92.67	0.000	.8309446	.8668533
60-70	1.009523	.0107486	93.92	0.000	.9884558	1.03059
70-80	1.19784	.0126163	94.94	0.000	1.173113	1.222568
80-90	1.484796	.0155048	95.76	0.000	1.454407	1.515185
90-95	1.911148	.0203231	94.04	0.000	1.871315	1.950981
95-100	4.137269	.1622893	25.49	0.000	3.819184	4.455353

```
. pshare histogram, yline(1) byopts(ti(Income distribution by age group))
```

Analysis of subpopulations



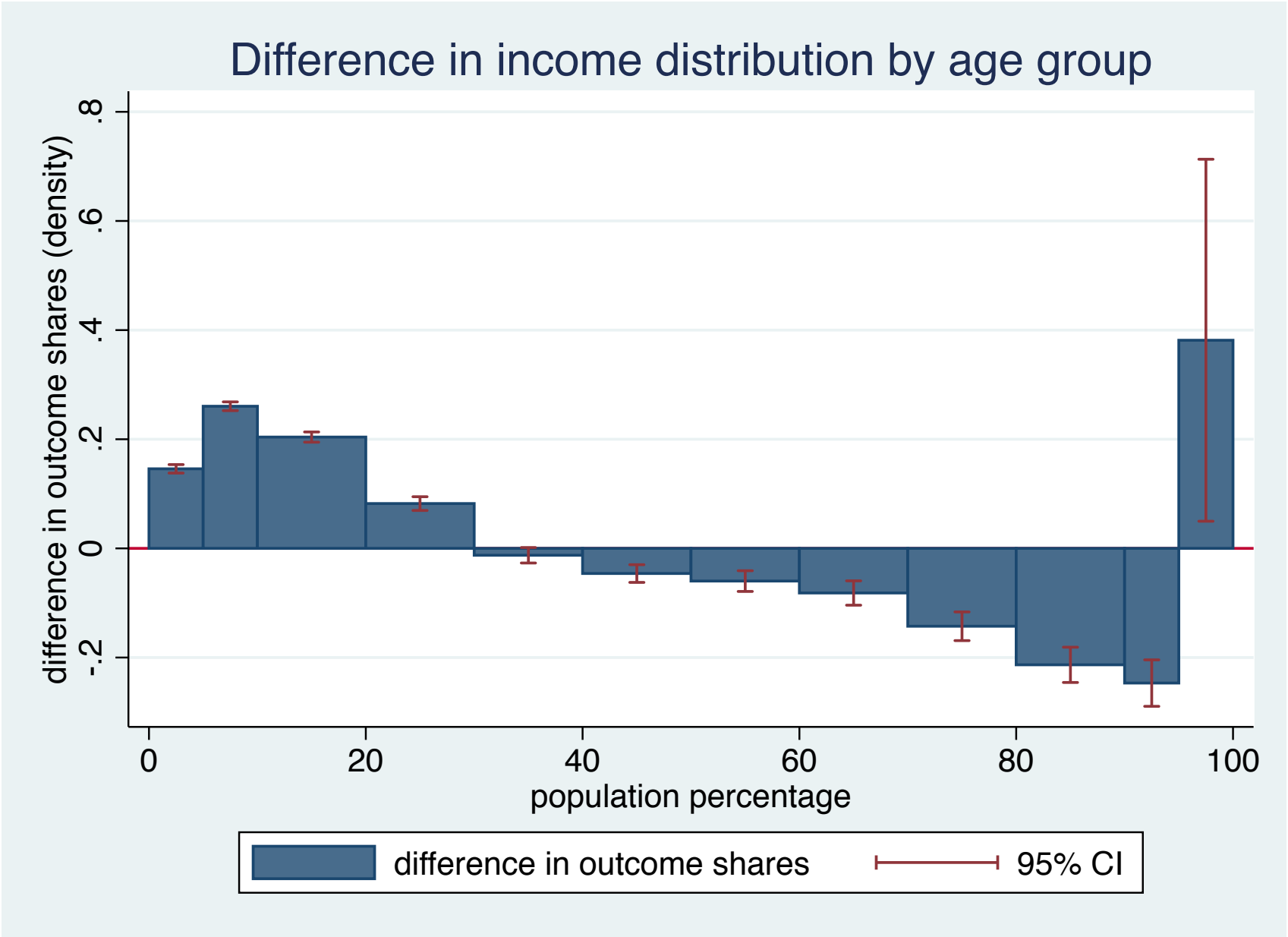
Subpopulation contrasts

```
. pshare contrast
Differences in percentile shares (density)      Number of obs     =    119,939
           15: agecat = 15-64
           65: agecat = 65 and over
```

income	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
65						
0-5	.1457933	.0039661	36.76	0.000	.1380198	.1535667
5-10	.2604233	.004082	63.80	0.000	.2524226	.268424
10-20	.2039867	.0047798	42.68	0.000	.1946184	.213355
20-30	.0821061	.0064225	12.78	0.000	.069518	.0946941
30-40	-.0126185	.0072372	-1.74	0.081	-.0268034	.0015664
40-50	-.0461002	.0082813	-5.57	0.000	-.0623314	-.029869
50-60	-.0598907	.009688	-6.18	0.000	-.078879	-.0409023
60-70	-.0817888	.0113809	-7.19	0.000	-.1040952	-.0594824
70-80	-.1427025	.0133956	-10.65	0.000	-.1689575	-.1164474
80-90	-.213378	.0164921	-12.94	0.000	-.2457023	-.1810537
90-95	-.2468179	.0217313	-11.36	0.000	-.2894109	-.2042248
95-100	.3813731	.1692052	2.25	0.024	.0497337	.7130125

```
(contrasts with respect to preceding subpopulation)
. pshare hist, yline(0) ti(Difference in income distribution by age group)
```

Subpopulation contrasts



Bivariate analysis: Wealth by income group

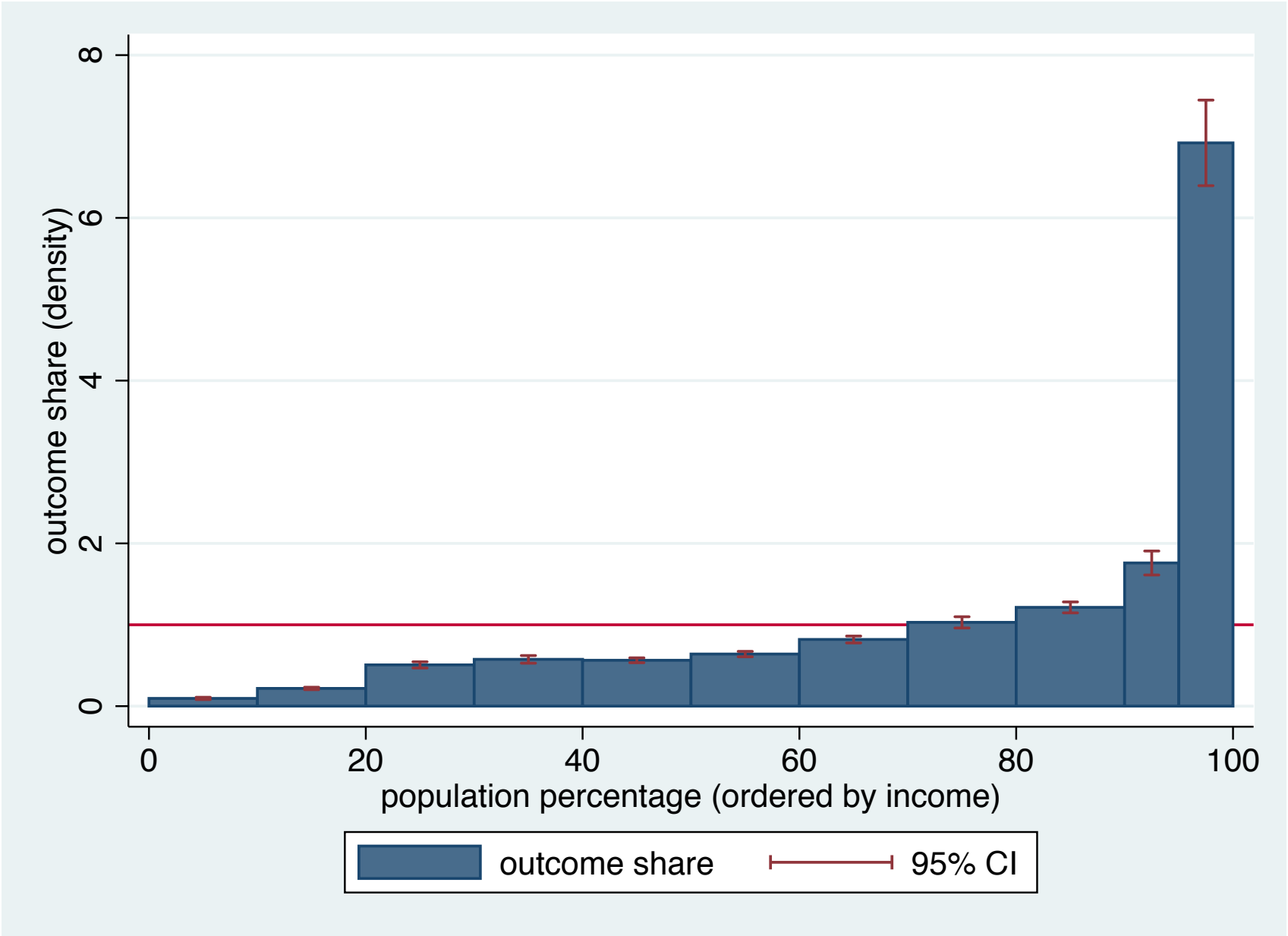
```
. pshare estimate wealth, p(10(10)90 95) pvar(income) density
Percentile shares (density)                     Number of obs     =    119,939
```

wealth	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
0-10	.094965	.0071814	13.22	0.000	.0808896	.1090403
10-20	.2175728	.0074677	29.14	0.000	.2029363	.2322093
20-30	.506814	.0192759	26.29	0.000	.4690335	.5445945
30-40	.5747378	.0238753	24.07	0.000	.5279426	.621533
40-50	.5637113	.0153038	36.83	0.000	.5337161	.5937066
50-60	.6399473	.0169505	37.75	0.000	.6067245	.67317
60-70	.8189444	.0215458	38.01	0.000	.776715	.8611737
70-80	1.029533	.0352054	29.24	0.000	.9605311	1.098535
80-90	1.213529	.0341494	35.54	0.000	1.146597	1.280461
90-95	1.75912	.0751482	23.41	0.000	1.611831	1.906409
95-100	6.921371	.2682702	25.80	0.000	6.395566	7.447176

```
(percentile groups by order of income)
. pshare histogram, yline(1)
```

The results show that the top income households are also the ones among which most of the wealth is accumulated.

Bivariate analysis: Wealth by income group



Problems

- Percentile shares are affected by small sample bias.
- The top percentile share is typically underestimated. This is because for unbound variables there is always some probability mass outside the range of the sampled data. A much larger outcome share is missed at the top than at the bottom.
- The problem is difficult to fix.
  - Corrections could be derived based on parametric assumptions.
  - Smoothing out the data by adding random noise can be an option, but this also requires parametric assumptions.
  - In `pshare`, I implemented non-parametric small-sample correction using a bootstrap approach: the bias in bootstrap samples is used to derive correction factors for the main results.
  - This works very well in terms of removing bias (unless the distribution is extremely skewed).
  - **However:** MSE increases compared to uncorrected results!
  - No idea how to improve on this. Therefore, small-sample correction is currently **not documented**.

References

- Ecoplan (2004). Verteilung des Wohlstands in der Schweiz. Bern: Eidgenössische Steuerverwaltung.
- Binder, D. A., M. S. Kovacevic (1995). Estimating Some Measures of Income Inequality from Survey Data: An Application of the Estimating Equations. Survey Methodology 21(2): 137-145.